

Inverse Symmetry Breaking on the lattice: an accurate MC study

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February 1, 2008

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Abstract

We present here a new MC study of ISB at finite temperature in a $Z_2 \times Z_2$ $\lambda\phi^4$ model in four dimensions. The results of our simulations, even if not conclusive, are favourable to ISB. Detection of the effect required measuring some critical couplings with six-digits precision, a level of accuracy that could be achieved only by a careful use of FSS techniques. The gap equations for the Debye masses, resulting from the resummation of the ring diagrams, seem to provide a qualitatively correct description of the data, while the simple one-loop formulae appear to be inadequate.

PACS: 05.70.Jk, 11.10.Wx, 98.80.Cq

Keywords: Finite temperature, Phase transitions, Inverse symmetry breaking, Symmetry non-restoration.

DFTUZ preprint 98/27 Napoli preprint 7/99 hep-lat/9903027

1 Introduction

In 1974, in his classic paper on thermal field theory, Steven Weinberg, quoting S. Coleman, reported on the unusual behavior of certain particle theory models at high temperatures. Using the example of a simple $O(N_1) \times O(N_2)$ scalar ϕ^4 model, he showed that it is possible to induce spontaneous symmetry breaking at arbitrarily high temperatures. He named this funny phenomenon Inverse Symmetry Breaking (ISB) or Symmetry-Non-Restoration (SNR), depending on whether the ground state was disordered or ordered at zero temperature. In order to make this seemingly paradoxical statement easier to accept, he observed that in Nature there is a substance, the Rochelle salt, that does exhibit this sort of inverse behavior. Its crystals are orthorhombic below the lower Curie point (-18°C), and monoclinic above it, which means that the crystal has a *larger* symmetry group at the *lower* temperature! Of course, Weinberg's statement was much stronger than this example, for he claimed that order could persist for ever, no matter how high the temperature, while the Rochelle salt, if heated enough, eventually melts.

In any case, ISB and SNR would have probably remained relegated among the amenities of field theory, if it were not for the fact that this mechanism can be implemented in realistic particle models and then applied to important cosmological issues related to the thermal history of the Early Universe. This is not the right place to discuss these very interesting applications of ISB and SNR, but to give the reader an idea of the number and relevance of the issues that have been treated, we just make a list and quote some references. Following the historical order, ISB and SNR were applied to: the breaking of CP symmetry [2], the monopole problem [3], baryogenesis [4], inflation [5], the breaking of P, strong CP and Peccei-Quinn symmetries [6] and finally supersymmetry [7].

The potential importance of ISB and SNR for cosmology induced several authors to study these phenomena more accurately, than was done in Weinberg's original paper. His analysis relied, in fact, on a simple one-loop computation, and so it was natural to question if higher order corrections would have changed the scenario. This was not an idle question for at least two reasons. On one side, it is well known that at high temperature the reliability of perturbation theory is questionable, because powers of the temperature can compensate for powers of the coupling constants, spoiling the expansion. On the other side, and maybe more importantly, when one tries

to induce ISB or SNR in realistic particle models, one is generally forced to consider rather large couplings in the scalar sector, in order to overcome the action of the gauge fields, which always work in favour of symmetry restoration at high temperature, and thus it becomes important to explore the robustness of ISB and SNR with respect to the coupling strengths.

The problem was attacked using a variety of approaches: large N expansions [8], the Gaussian effective potential [9], the constraint effective potential [10], gap equations [11], the average effective action [12] and recently chiral effective Lagrangians [13]. The results were contradictory: some authors concluded that ISB and SNR are artifacts of the one-loop approximation, while others confirmed the existence of these phenomena, but observed that higher order-corrections have the effect of reducing the size of the parameter region for which they can occur.

This state of things motivated us to start a program of MC simulations to study ISB, which seemed to us the only way to carry a fully non-perturbative analysis of this problem. We chose the simplest model that can exhibit ISB, namely a $Z_2 \times Z_2$ two-scalars ϕ_d^4 theory. In the first run, we simulated the model in three dimensions [14] and afterwards we followed with four [15]. In both cases we found no sign of ISB, despite the fact that, for the values of the coupling constants that were simulated, the one-loop conditions for ISB were strongly satisfied. Recently, the authors of ref.[16] examined a $Z_2 \times O(4)$ two-scalar ϕ^4 model, in four dimensions, and claimed to have found a clear evidence that ISB was taking place.

In this paper we present the results of a new series of simulations of our $Z_2 \times Z_2$ model. The strategy that we have followed is essentially the same of our previous paper [15]. Our two scalar model depends on five parameters, three coupling constants λ_1, λ_2 and λ , and two hopping parameters κ_1 and κ_2 . We fixed once and for all the values of the coupling constants: with respect to [15], this time we enforced the perturbative conditions for ISB (in the direction of the field ϕ_2) much more strongly and at the same time we took smaller values in order to be closer to the perturbative region. We then studied the phase diagram as a function of the two hopping parameters. In the (κ_1, κ_2) plane we found transition lines of first and second order, but since we were interested in ISB, we focused our efforts on the critical line for the breaking of the field ϕ_2 . This line is roughly parallel to the κ_1 axis and the aim was to determine in what direction it shifts, when the temperature is increased: ISB requires that, for $T > 0$, it shifts towards smaller values of

κ_2 , at least in a neighbourhood of the scaling region.

As it is well known, finite temperatures are simulated by lattices with a finite extension N_t in one direction, the temperature being $T = 1/(N_t a)$, with a the lattice spacing. What made the simulations very hard is the extreme smallness of the effect that we show in this paper: even at the highest temperature, which corresponds to $N_t = 2$, in order to detect the shift reliably, it was necessary to measure the critical values of κ_2 , both for $T = 0$ and for $T > 0$, with six significant digits! As it is well known, the only safe way to achieve such a huge precision is via an accurate analysis of Finite Size Scaling (FSS). This required large lattices and tremendous statistics: in one case, for example, we simulated a 20^4 lattice, with 8×10^6 iterations.

For $N_t = 2$, the direction of the shift was favourable to ISB, but we cannot consider this result as conclusive. Due to the difficulty of the measurements, we could not simulate larger values of N_t , as it is necessary in order to make sure, via scaling analysis, that things would go in the same way for all N_t 's.

As a check, we compared the theoretical predictions with the MC value of the critical temperature for which we observed ISB. While the simple one-loop estimate is grossly incorrect, we found a reasonable agreement with the value obtained from the gap equations, which result from resumming the ring diagrams of the perturbative series [17]. These equations seem to give a qualitatively correct description of the MC data, and explain as well why we did not find ISB in our previous simulations in [15].

We close this introductory Section with a plan of the paper: in Section 2 we introduce our lattice model and discuss its phase diagram at $T = 0$. In Section 3 we review the perturbative picture of ISB, while in Section 4 we discuss in detail our strategy to study ISB on the lattice. In Section 5 we discuss how the results of the simulations compare with the theoretical predictions, while Section 6 contains a discussion of the recent MC studies in [15] and [16]. Finally, Section 7 contains our concluding remarks.

2 The model and its lattice formulation

We consider the theory for two real scalar fields in 4 euclidean dimensions, described by the bare (euclidean) action:

$$S = \int d^4x \left\{ \sum_{i=1,2} \left[\frac{1}{2} (\partial_\mu \Phi_i^{(0)})^2 + \frac{1}{2} \overline{m}_i^{(0)2} \Phi_i^{(0)2} + \frac{g_i^{(0)}}{4!} \Phi_i^{(0)4} \right] + \frac{1}{4} g^{(0)} \Phi_1^{(0)2} \Phi_2^{(0)2} \right\}. \quad (1)$$

What will be essential for ISB and SNR, in the above action the quartic bare coupling $g^{(0)}$ can be *negative*. If $g^{(0)} > 0$ the condition of boundedness from below of the potential is satisfied if:

$$\begin{aligned} g_1^{(0)} &> 0, \quad g_2^{(0)} > 0, \\ g_1^{(0)} g_2^{(0)} &> 9 g^{(0)2}. \end{aligned} \quad (2)$$

When regularized on an infinite four-dimensional cubic lattice of points Ω with lattice spacing a , the above action is replaced by its discretized version

$$\begin{aligned} S_L = \sum_{x \in \Omega} a^4 \left\{ \sum_{i=1,2} \left[\sum_{\mu} \frac{1}{2} (\Delta_{\mu}^{(a)} \Phi_{i,L})^2(x) + \frac{1}{2} \overline{m}_i^{(0)2} \Phi_{i,L}^2(x) + \frac{g_i^{(0)}}{4!} \Phi_{i,L}^4(x) \right] + \right. \\ \left. + \frac{g^{(0)}}{4} \Phi_{1,L}^2(x) \Phi_{2,L}^2(x) \right\}, \end{aligned} \quad (3)$$

where $\Delta_{\mu}^{(a)}$ is the lattice derivative operator in the direction μ :

$$\Delta_{\mu}^{(a)} \Phi_{i,L}(x) \equiv \frac{\Phi_{i,L}(x + \hat{\mu}a) - \Phi_{i,L}(x)}{a}. \quad (4)$$

We find it convenient to measure all dimensionful quantities in (3) in units of the lattice spacing; thus we define:

$$\phi_{i,L}(x) \equiv a \Phi_{i,L}(x), \quad m_i^{(0)2} \equiv a^2 \overline{m}_i^{(0)2}. \quad (5)$$

In terms of the dimensionless quantities the lattice action now reads:

$$\begin{aligned} S_L = \sum_{x \in \Omega} \left\{ \sum_{i=1,2} \left[\sum_{\mu} \frac{1}{2} (\Delta_{\mu}^{(1)} \phi_{i,L})^2(x) + \frac{1}{2} m_i^{(0)2} \phi_{i,L}^2(x) + \frac{g_i^{(0)}}{4!} \phi_{i,L}^4(x) \right] + \right. \\ \left. + \frac{g^{(0)}}{4} \phi_{1,L}^2(x) \phi_{2,L}^2(x) \right\}, \end{aligned} \quad (6)$$

The standard lattice notation is obtained with a further redefinition of the fields and couplings in (6) according to:

$$\phi_{i,L}(x) = \sqrt{\kappa_i} \phi_{i,r}, \quad g_i^{(0)} = \frac{24\lambda_i}{\kappa_i^2}, \quad m_i^{(0)2} = 2 \frac{1 - 2\lambda_i - 4\kappa_i}{\kappa_i} \quad (7)$$

and

$$g^{(0)} = \frac{4\lambda}{\kappa_1 \kappa_2}. \quad (8)$$

After these redefinitions we get our final form of the lattice action

$$S_L = \sum_{r \in \mathbf{Z}^4} \left\{ \sum_{i=1,2} \left[-\kappa_i \sum_{\mu} \phi_{i,r} \phi_{i,r+\hat{\mu}} + \lambda_i (\phi_{i,r}^2 - 1)^2 + \phi_{i,r}^2 \right] + \lambda \phi_{1,r}^2 \phi_{2,r}^2 \right\}. \quad (9)$$

For generic values of the parameters, the action has a $\mathbf{Z}_2 \times \mathbf{Z}_2$ symmetry which can be spontaneously broken. The model is expected to have only one fixed point, the Gaussian one, corresponding to $m_1^{(0)2} = m_2^{(0)2} = g_1^{(0)} = g_2^{(0)} = g^{(0)} = 0$ (or $\kappa_1 = \kappa_2 = 1/4$ and $\lambda_1 = \lambda_2 = \lambda = 0$), which has a null attractive domain in the infrared. In $4 - \epsilon$ dimensions there exist five more fixed points: 1) the Heisenberg fixed point, for $a^\epsilon g_1^{(0)} = a^\epsilon g_2^{(0)} = 3a^\epsilon g^{(0)} = g_H^*$ and $m_1^{(0)2} = m_2^{(0)2} = m_H^{2*} < 0$, where the symmetry of the model is enhanced to $O(2)$. 2) the three Ising fixed points, for $g^{(0)} = 0$ and $a^\epsilon g_i^{(0)} = g_I^*$, $m_i^{(0)2} = m_I^{2*}$ for some i , where the model splits into two independent ϕ_4^4 models. 3) the Cubic fixed point, for $a^\epsilon g_i^{(0)} = a^\epsilon g^{(0)} = g_I^*/2$, $m_i^{(0)2} = m_I^{2*}$, which again splits into two independent ϕ_4^4 models, after a $\pi/4$ -rotation of the fields.

The phase diagram of the model (9) (at $T = 0$), for fixed values of λ_1 , λ_2 and λ , is shown in Fig. 1.

There are four distinct phases separated by four critical lines A, B, D and E: the disordered one corresponds to the lower left corner of the picture, while the upper right corner represents the totally ordered phase. The wedges in between them are partially ordered phases: in the left wedge the field ϕ_2 is ordered, while ϕ_1 is disordered; in the right wedge it occurs the contrary.

If there was no interaction between ϕ_1 and ϕ_2 , we would have two independent ϕ_4^4 models and then the critical lines would all be of second order. The lines A and E would form a straight line parallel to the κ_1 axis, and similarly the lines B and D would form a straight line parallel to the κ_2 axis. The presence of the interaction bends them as can be seen in the Figure. We

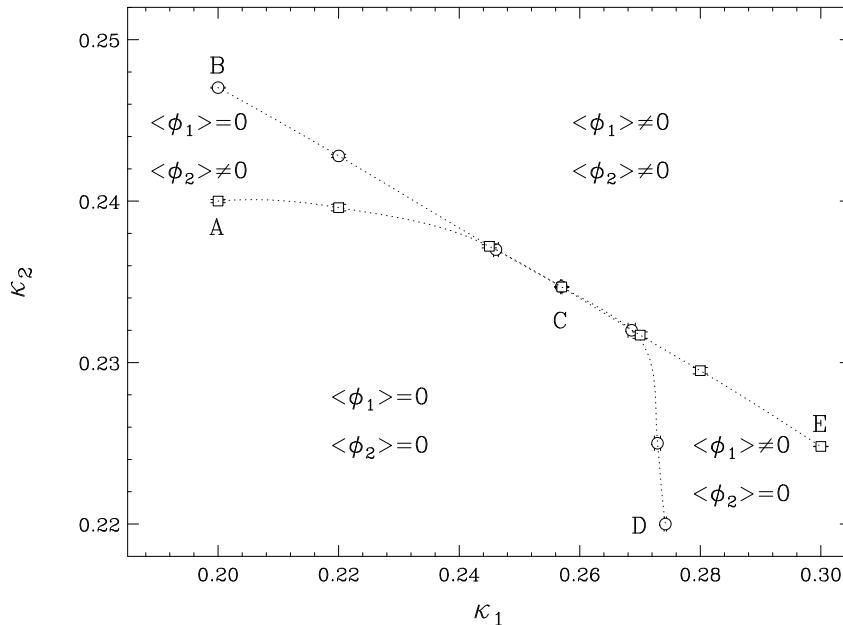


Figure 1: Phase diagram at $\lambda_1 = 0.3375$, $\lambda_2 = 0.01125$, $\lambda = -0.112$.

managed to determine the order of the critical lines only at some distance from the region C, where the four lines merge together. The critical line A is the only one that we studied accurately: away from C, it is of second order and we observed for it Gaussian critical exponents, as it had to be expected from triviality. As for the remaining critical lines, there is a clear evidence that D is of second order, while B and E appear to be of first order. Close to C, things become unclear: while on small lattices it becomes impossible to distinguish the various phase transitions, on large ones the onset of strong metastabilities prevented us from getting any indications at all.

Understanding the phase diagram in the neighbourhood of C is of course extremely important. An important issue to address is whether the four critical lines actually meet at a single point, a double critical point that we will call \mathcal{C} , and what is its order.

Indeed, the indications coming from perturbation theory suggest that, when g^R is *negative*, as required for ISB or SNR to occur, \mathcal{C} , if it exists, should be a first order critical point. This can be seen by integrating the one-loop renormalization group equations (RGE), in the scaling limit of large

correlation lengths ξ_i for both fields

$$\xi_i = c_i \xi, \quad \xi \gg 1,$$

where c_i are constants. In this limit the RGE of our model read:

$$\begin{aligned} \frac{dg_1^R}{dt} &= -\frac{3}{16\pi^2}[(g_1^R)^2 + (g^R)^2], \\ \frac{dg_2^R}{dt} &= -\frac{3}{16\pi^2}[(g_2^R)^2 + (g^R)^2], \\ \frac{dg^R}{dt} &= -\frac{1}{16\pi^2}(g_1^R + g_2^R + 4g^R)g^R, \end{aligned} \tag{10}$$

where $t = \log \xi$. Numerical integration of these equations shows that, if one starts from a set of initial values for the renormalized couplings with g^R negative, g_1^R and g_2^R are driven by g^R towards *negative* values in a finite interval of t , something that is generally believed to indicate a *first-order* phase transition. The fact that for negative values for the coupling g^R the double critical point really is of first order, is shown in ref.[18], where the phase diagram of our model is studied, using a suitable truncation of the exact non-perturbative flow equations, for the particular case $g_1^{(0)} = g_2^{(0)}$, $m_1^{(0)2} = m_2^{(0)2}$, when an extra $\phi_1 \leftrightarrow \phi_2$ symmetry is present (this case is not directly related to ISB or SNR because this extra symmetry excludes a priori the possibility of these phenomena).

If this picture of the double phase transition for $g^R < 0$ turns out to be correct, one may question whether in this case such a model can be used at all as an elementary particles theory. The answer to this question depends on the actual strength of the first-order phase transition. The fact that the correlation lengths of one or both fields remain finite near \mathcal{C} is not necessarily a problem per se. In fact, even if \mathcal{C} were of second order for both fields, triviality would prevent one from taking the correlation lengths to infinity anyway. Thus, the relevant question is to see if the correlation lengths become sufficiently large near \mathcal{C} , for the presence of the lattice cut-off to become negligible in low-energy amplitudes. This in turn depends on the absolute magnitude of the coupling constants. For small couplings, \mathcal{C} should be only weakly first order and one should be able to achieve large correlation lengths for both fields. In realistic particle models for ISB and SNR the problem may be more serious: the presence of gauge couplings, that work in favour

of symmetry restoration, usually forces one to take large negative couplings g^R , for ISB or SNR to occur [11], and this can then give rise to rather strong first order phase transitions in the scalar sector, thus spoiling the model.

3 High- T perturbation theory

In this Section we briefly review the predictions of perturbation theory on the high-temperature behavior of the model (1). We shall see that, for certain choices of the couplings, the symmetric vacuum of the theory becomes unstable at sufficiently high temperatures. In order to explore this phenomenon, we need to compute the leading high- T contribution to the second derivatives of the effective potential in the origin, or equivalently the 1PI 2-point functions for zero external momenta p . A standard one-loop computation (in the imaginary time formalism) [19] in our model gives the result:

$$\begin{aligned} M_1^2(T) &\equiv -\Gamma_1^{(2)}(p=0, T) = m_1^2 + \frac{T^2}{24} \left[g_1^R h\left(\frac{m_1}{T}\right) + g^R h\left(\frac{m_2}{T}\right) \right] , \\ M_2^2(T) &\equiv -\Gamma_2^{(2)}(p=0, T) = m_2^2 + \frac{T^2}{24} \left[g_2^R h\left(\frac{m_2}{T}\right) + g^R h\left(\frac{m_1}{T}\right) \right] , \end{aligned} \quad (11)$$

where m_i^2 represent the renormalized masses at $T=0$ and $h(x)$ is the function:

$$h(x) = \frac{6}{\pi^2} \int_0^\infty dy \frac{y^2}{(x^2 + y^2)^{1/2} (e^{(x^2 + y^2)^{1/2}} - 1)} . \quad (12)$$

It is clear that $h(x)$ is positive, monotonically decreasing and that it approaches zero when $x \rightarrow \infty$. For small values of x , $h(x)$ has the asymptotic expansion:

$$h(x) = 1 - \frac{3}{\pi}x - \frac{3}{2\pi^2}x^2 \left(\ln \frac{x}{4\pi} + \gamma - \frac{1}{2} \right) , \quad (13)$$

γ being the Euler constant. In the limit of very high-temperatures $m_i/T \ll 1$, eqs.(11) reduce to:

$$\begin{aligned} M_1^2(T) &= m_1^2 + \frac{T^2}{24} (g_1^R + g^R) , \\ M_2^2(T) &= m_2^2 + \frac{T^2}{24} (g_2^R + g^R) . \end{aligned} \quad (14)$$

Assuming for simplicity that the bare couplings are so small that they can be identified with the renormalized ones, and recalling that the coupling $g^{(0)} \approx g^R$ can be *negative*, it is easy to see that for g_1^R/g_2^R sufficiently large, the range (2) of stability for the potential includes values of g^R such that, say, $g_2^R + g^R$ is *negative* (it can be proven easily from eqs.(2) that one cannot have $g_1^R + g^R < 0$ at the same time). In this case we see from eq.(14) that $M_2^2(T)$ becomes negative at sufficiently high temperatures, irrespective of its value at $T = 0$. This is the essence of the phenomena of SNR and ISB: in multiscalar models some Debye masses can become negative and so one can have spontaneous symmetry breaking at arbitrarily high temperatures. We now focus on ISB, the phenomenon to be explored in this paper: in this case we take $m_1^2 > 0$, $m_2^2 > 0$, so that the vacuum is disordered at $T = 0$, and again assume that $g_2^R + g^R < 0$. If one starts increasing the temperature, it will happen that, at a certain critical temperature T_c , the field ϕ_2 will undergo spontaneous symmetry breaking and will stay broken at all higher temperatures. The field ϕ_1 , instead, will remain disordered. The critical temperature T_c can be estimated from eq.(14) to be:

$$\frac{T_c}{m_2} = \sqrt{-\frac{24}{g^R + g_2^R}} \quad . \quad (15)$$

Even though they capture the main features of ISB and SNR, at a closer inspection, eqs.(14) and (15) appear unsatisfactory in two respects. First of all, the condition for the high-T instability in, say, the ϕ_2 direction, $g_2^R + g^R < 0$, as deduced from the second of eqs.(14), does not involve g_1^R . Since the bound (2), together with the condition $g_2^R + g^R < 0$, typically implies a hierarchy among the couplings $g_1^R \gg |g^R| \geq g_2^R$, it is clear that next-to-leading order corrections involving g_1^R can produce significant corrections to the lowest order result. The second fault of eqs.(14) and (15) is the most important for our study of ISB. It is the fact that the estimate (15) of the critical temperature does not involve m_1^2 : this looks physically incorrect, for one can imagine that for $m_1 \gg m_2$ the field ϕ_1 should decouple from ϕ_2 for temperatures $T \ll m_1$ and so ISB or SNR should not occur before the scale m_1 is reached. Both limitations of the one-loop picture can be overcome by performing the resummation of the so called ring diagrams.

As it is well known [17], these diagrams represent the dominant next-to-leading corrections at high temperatures and their inclusion leads to a more

accurate description of both the phase transition and the high temperature behavior of the thermal masses. This infinite resummation leads to self-consistent gap equations for the Debye masses $M_i^2(T)$, which in our case read

$$\begin{aligned} x_1^2 &= \frac{m_1^2}{T^2} + \frac{g_1^R}{24}h(x_1) + \frac{g^R}{24}h(x_2) \ , \\ x_2^2 &= \frac{m_2^2}{T^2} + \frac{g_2^R}{24}h(x_2) + \frac{g^R}{24}h(x_1) \ , \end{aligned} \quad (16)$$

where we defined $x_i = M_i(T)/T$. For any given temperature T , the symmetric vacuum is stable if the above equations admit real (positive) solutions for x_i . At sufficiently low temperatures, this is clearly the case if we take $m_1^2 > 0$, $m_2^2 > 0$. One can now gradually increase T and follow the evolution of x_1 and x_2 : if one of them, say x_2 , approaches zero at $T = T_c$, and if for $T > T_c$ the gap equations do not admit anymore real solutions, one can argue that for $T > T_c$ there will be ISB in the ϕ_2 direction. It is clear that this can happen only if g^R is negative and its absolute value is large enough in comparison with g_2^R . A detailed study of the conditions required is given in the first of refs.[11], where it is shown that the parameter region for which ISB or SNR occur is indeed smaller than that predicted by the naive eqs.(14). An analogous result was later confirmed by studies based on different non-perturbative techniques [12].

Assuming now that at high temperatures there develops an instability in the ϕ_2 direction, a better estimate of the critical temperature T_c , than that given by the one-loop equations (15), can be obtained by setting $x_2 = 0$ in eqs.(16) and then solving the system:

$$\begin{aligned} x_1^2 &= \frac{m_1^2}{T_c^2} + \frac{1}{24}(g_1^R h(x_1) + g^R) \ , \\ 0 &= \frac{m_2^2}{T_c^2} + \frac{1}{24}(g_2^R + g^R h(x_1)) \ , \end{aligned} \quad (17)$$

where we have used $h(0) = 1$. Upon comparing the second of eqs.(17) with the second of eqs.(14), it is clear that the inclusion of the ring diagrams pushes the phase transition to higher temperatures. This is so because the negative coupling g^R , which is the cause of ISB, comes with the factor $h(x_1)$, which is always less than one. Since x_1 is larger then m_1^2/T_c^2 , as it is clear

from the first of eqs.(17), we see that a large mass m_1^2 will make ISB harder to achieve. In fact, the influence of m_1^2 can be so strong as to cause the disappearance of the symmetry-breaking phase transition predicted by the one-loop formula. As we shall explain in greater detail in Sec. 6, this effect probably explains why we did not observe ISB in our previous work [15].

4 ISB on the lattice

We saw in the previous Section that, for $g_2^R + g^R$ sufficiently negative, continuum perturbation theory predicts ISB for the ϕ_2 field. Even if the mass parameters are chosen in such a way that the ground state is disordered at $T = 0$, the field ϕ_2 should develop a non-vanishing vev above some critical temperature T_c and should remain ordered at all higher temperatures. The field ϕ_1 , instead, should remain disordered.

In principle it is very simple to study ISB on the lattice. As it is well known, finite temperatures are simulated by lattices with a finite extension in the “time” direction (and infinite extension in the remaining “space” directions, as required by the thermodynamic limit), the temperature being related to the number N_t of sites by the relation:

$$T = \frac{1}{N_t a} , \quad (18)$$

with a the lattice spacing. So, in order to study ISB, one should simply check how the critical lines of the $T = 0$ phase diagram, shown in Fig.1, shift in the (κ_1, κ_2) plane (for fixed values of λ_1 , λ_2 and λ) as a function of the temperature, which means concretely as a function of N_t (in principle, one could vary the temperature also by varying the lattice spacing in the time direction, but we have not used this method). Since we are specifically interested in the behavior of the field ϕ_2 , it is sufficient to focus our attention on the critical line of this field, the line A-E of Fig.1. If, for the selected values of λ_1 , λ_2 and λ , ISB occurs, it should happen that, when the temperature is increased, namely when N_t is diminished, the ϕ_2 -critical line penetrates more and more deeply in the disordered region of the $T = 0$ phase diagram, at least in a neighbourhood of the double critical point \mathcal{C} . The effect should be clearest at the highest temperatures, i.e. for the smallest values of N_t . To help the reader visualize the situation, we have shown in Fig.2 the expected

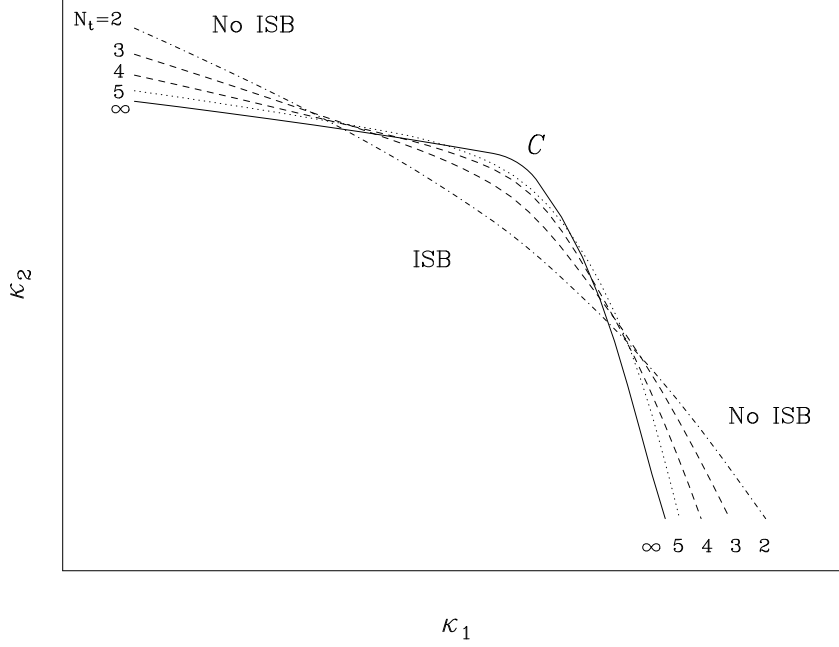


Figure 2: Expected (qualitative) temperature-behavior of the critical line for the ϕ_2 transition, in case of ISB. The region studied by us is the one to the left of C .

N_t -behavior of the ϕ_2 -critical line, in case of ISB. Some comments are in order.

We expect that far from the double critical point C the N_t behavior of the ϕ_2 -critical line should always be the normal one, namely the critical line should move towards greater values of κ_2 when N_t is diminished. This conjecture looks reasonable, if one considers that far from C , the field ϕ_1 should be far from critical and its correlation length should become very small. This implies that the influence of the field ϕ_1 over the field ϕ_2 , near the critical line of the latter, should be negligible and so the behavior of ϕ_2 should be that of a normal ϕ^4 theory, where an increase of temperature shifts the transition upwards. This means that ISB should be visible, if it happens at all, only close enough to C , in the scaling region where the correlation lengths of both fields become large. This explains why, in Fig. 2, we have drawn the critical lines for $N_t = 2, 3, 4, 5$ below that of the $T = 0$ theory ($N_t = \infty$) only near C , while away from C they all lie above it.

Such a behavior poses a very serious problem: imagine that for the few values of N_t simulated, one has found a result like that shown in Fig. 2, which supports ISB. How can one convince oneself that nothing will change for greater values of N_t ? One could suspect that for N_t large enough the critical line might pass entirely above that of the $T = 0$ theory, thus leading to the disappearance of ISB. In practice, it is then very important to have a criterion to decide if, whatever behavior is observed with the few N_t 's that are simulated, it will not change when going to higher N_t 's. The common practice to address this question is to search for scaling in the N_t dependence of some observable: this is what we did in [15], where, for a fixed value of κ_1 and several values of N_t , we measured the critical values of κ_2 , $\kappa_2^c(N_t)$. The values we got were all greater than the critical value κ_2^c for $T = 0$, a result against ISB (very likely, the value of κ_1 used for the simulations in [15] was too far from \mathcal{C} , for ISB to be visible. We were probably in the region of Fig. 2 on the left of \mathcal{C} , where the behavior of the ϕ_2 critical line becomes the “normal” one. We shall have more to say on this in Sec. 6.) . The good scaling of $\kappa_2^c(N_t)$ with N_t convinced us that no change would have occurred for larger N_t . We think that applying this method to make sure that ISB survives for all values of N_t would be very difficult, because we believe that in order to observe ISB for a number of values of N_t large enough for a scaling analysis to be possible, one needs look very close to the critical point \mathcal{C} , where the large correlation lengths make it necessary to use very large lattices for a reliable measurement of the critical couplings.

As a matter of fact, in the new series of simulations presented here, we have been able to simulate only one value of N_t , $N_t = 2$, and we needed a very large statistics in order to detect reliably the shift of the ϕ_2 critical line.

Let us briefly explain the method we have followed, postponing the details to the next two subsections.

The first part of the method is exactly the same as that of ref.[15], but it is useful to review it here again. We fixed once and for all a set of values for λ_1, λ_2 and λ . This time, we took $\lambda_1 = 0.16875$, $\lambda_2 = 0.0001125$, $\lambda = -0.00784$. There are two important differences with ref.[15]:

- a) the new values of the couplings are smaller than the old ones [15]. This was done to be closer to the perturbative region, but it increased tremendously the difficulty of the simulations, because the shift of the critical line with N_t was this time much smaller than before;
- b) more importantly, the ratio $|\lambda|/\lambda_2$ is now much greater than before (more

N_t	N_s	# runs	Thermalization sweeps	# sweeps
2	6	1	5×10^4	4×10^6
2	8	1	10^5	8×10^6
2	10	1	2×10^5	10^7
2	12	1	3×10^5	10^7
2	16	2	2×10^5	8×10^6
2	20	4	8×10^4	6.4×10^6
6	6	1	5×10^4	4×10^6
8	8	1	10^5	8×10^6
10	10	1	2×10^5	4×10^6
12	12	1	3×10^5	8×10^6
16	16	4	2×10^5	9.6×10^6
20	20	6	2.4×10^5	9.6×10^6

Table 1: Statistics for the simulations on lattices of sizes $N_t \times N_s^3$.

or less 70 against 10), and so there is now a much stronger push towards ISB.

After a quick analysis of the phase diagram at $T = 0$, we selected a value of κ_1 , $\kappa_1 = 0.24$, to the left of \mathcal{C} , and then we measured accurately the corresponding critical value κ_2^c of κ_2 along the ϕ_2 critical line, always at $T = 0$.

Near this point, we measured the $T = 0$ values of the renormalized couplings and the correlation lengths of both fields, for the same value of κ_1 , but for a value of κ_2 slightly less than κ_2^c , in the disordered phase (the results of these measurements are given in Table 2). The value of $\xi_1 \simeq 1$ that we got is not very large, but still acceptable for us to believe that we were probing the continuum region. In any case, we could not achieve larger values for ξ_1 , because near to \mathcal{C} we encountered strong metastabilities that made any simulations impossible. We notice also that the value of g_1^R is quite large and not very accurate; on the contrary, what we think is the essential, we see that g_2^R and $|g^R|$ are both very small and that the condition for ISB is implemented very strongly, since $|g^R|/g_2^R \simeq 10$. The one-loop formulae obviously predict that ISB should occur for these values of the couplings, and it can be checked that the ratio $|g^R|/g_2^R$ is large enough for ISB to survive also

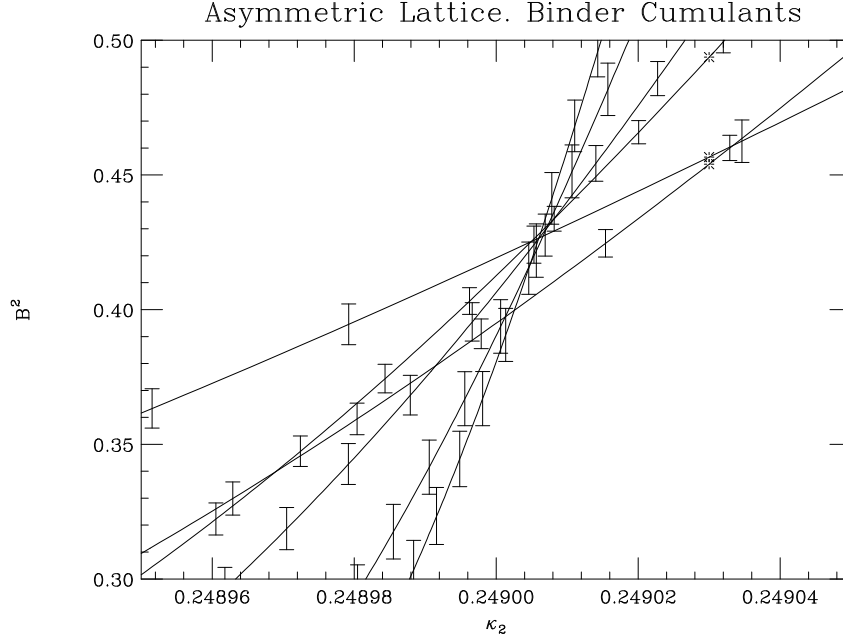


Figure 3: Binder cumulants for the $2 \times N_s^3$ lattices ($N_s = 6, 8, 10, 12, 16, 20$) at $\lambda_1 = 0.16875$, $\lambda_2 = 0.0001125$, $\lambda = -0.00784$ and $\kappa_1 = 0.24$.

the inclusion of the ring diagrams. This convinced us that we had found a good simulation region.

Having performed all these measurements at $T = 0$, we turned to finite values of N_t . Due to the extreme smallness of the effect, we were able to detect the shift of κ_2^c (for $\kappa_1 = 0.24$) only for the smallest value of N_t , $N_t = 2$. Even in this case, the shift was of the order of one part in hundred thousands and so larger values on N_t look out of reach. Since this shift turned out to be towards smaller values of κ_2 , our result can be considered as favourable to ISB.

In the next two Subsections, we describe the details of the numerical simulations and how we managed to measure the critical values of κ_2 with the high accuracy that was needed.

4.1 MC Simulation: determination of κ_2^c

For this new simulation, we used the Metropolis Algorithm (cluster was not efficient), with 3 hits and acceptance around 60%. We simulated lattices of sizes $2 \times N_s^3$ and N_s^4 , with $N_s = 6, 8, 10, 12, 16, 20$. In the new region of parameters the field fluctuations are strong and the transitions for different lattice sizes are very close. We had to accumulate very large statistics in order to obtain accurate results. In Table 1 we summarize the statistics for every lattice: number of runs, sweeps left for thermalization and total number of sweeps for every run.

As we said above, all our simulations were performed for a single value of κ_1 , $\bar{\kappa}_1=0.24$. A fast simulation on the 6^4 lattice gave us the approximate position, $\kappa_2 = 0.24903$, of the phase transition of the field ϕ_2 (the field ϕ_1 remains disordered across the transition). All the subsequent massive simulations were performed at this point, and the results were extrapolated in a narrow κ_2 -interval around it by means of the Spectral Density Method [22] (SDM). The observable used to locate with high precision the phase transition was the Binder cumulant:

$$U_i(N_s, \kappa_1, \kappa_2) = \frac{3}{2} - \frac{\langle M_i^4 \rangle}{2\langle M_i^2 \rangle^2}, \quad (19)$$

where M_i stands for the magnetization of the field ϕ_i .

Figs.3 and 4 show the values of U_2 obtained by extrapolating the results of the simulations by means of the SDM. Every pair of curves (N_s and \hat{N}_s) determines a crossing point, that gives an estimate $\kappa_2^*(N_s, \hat{N}_s)$ of the critical point κ_2^c . These values do not suffer from finite size effects, except for corrections to scaling, which can be parametrized and used to obtain κ_2^c , according to the scaling law:

$$\kappa_2^*(N_s, bN_s) - \kappa_2^c = \frac{1 - b^{-\omega}}{b^{1/\nu} - 1} N_s^{-\omega-1/\nu}, \quad (20)$$

where ω is the exponent for the corrections to scaling. In the calculation of the error, due account was taken of the fact that the pair crossings were not all independent of each other (out of the $N(N-1)/2$ crossings between N curves, only $2N-3$ are independent of each other).

As for the exponents occurring in eq. (20), we had to distinguish the symmetric (N_s^4) lattices from the asymmetric ($2 \times N_s^3$) ones. In the case of

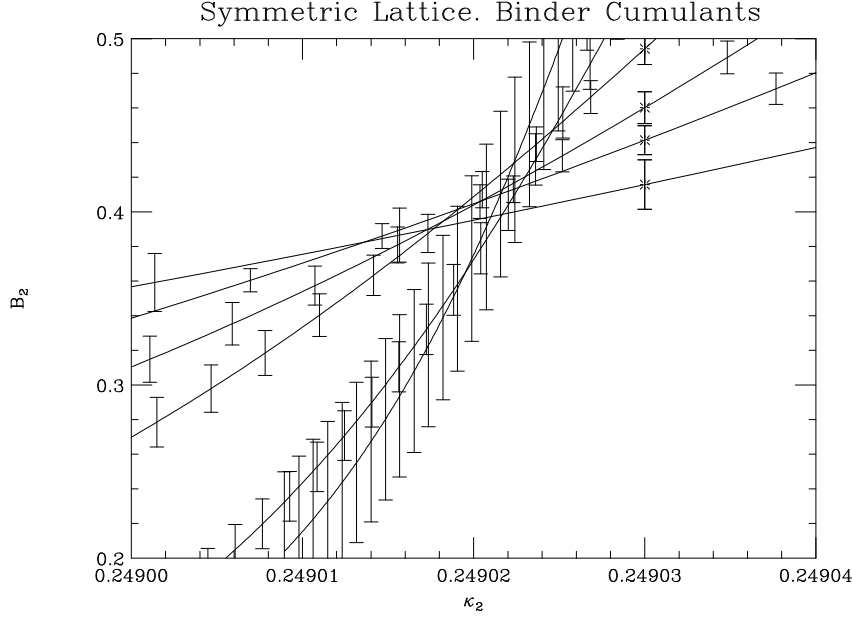


Figure 4: Binder cumulants for the N_s^4 lattices ($N_s = 6, 8, 10, 12, 16, 20$) at $\lambda_1 = 0.16875$, $\lambda_2 = 0.0001125$, $\lambda = -0.00784$ and $\kappa_1 = 0.24$.

the latter, since the scaling parameter is N_s , while N_t is fixed, according to the hypothesis of dimensional reduction and universality, we used the exponents of the Ising model in *three* dimensions, namely $\nu = 0.63$ and $\omega = 0.8$. As a check, we also computed ν directly from the data obtaining values fully compatible with the above one (see Fig. 7).

In the case of the symmetric lattices, we used the mean field exponents $\omega = 0, \nu = 1/2$. As seen from eq.(20), these values imply that, apart from logarithmic corrections, all the crossings should occur for the same value of κ_2 , and this is approximately what we found. A better estimate of κ_2^c was then obtained by setting $\omega \neq 0$ in eq.(20) and taking the limit, for $\omega \rightarrow 0$, of the corresponding values of κ_2^c . In the process, we observed a very fast convergence.

The result of the extrapolation for $N_s \rightarrow \infty$ is shown in Fig. 5. We got $\kappa_2^c(N_t = 2) = 0.249005(2)$ and $\kappa_2^c(T = 0) = 0.249022(2)$. This means that the disordered region diminishes when T is increased, as ISB requires, at least for the case of the highest temperature reachable on the lattice ($N_t = 2$).

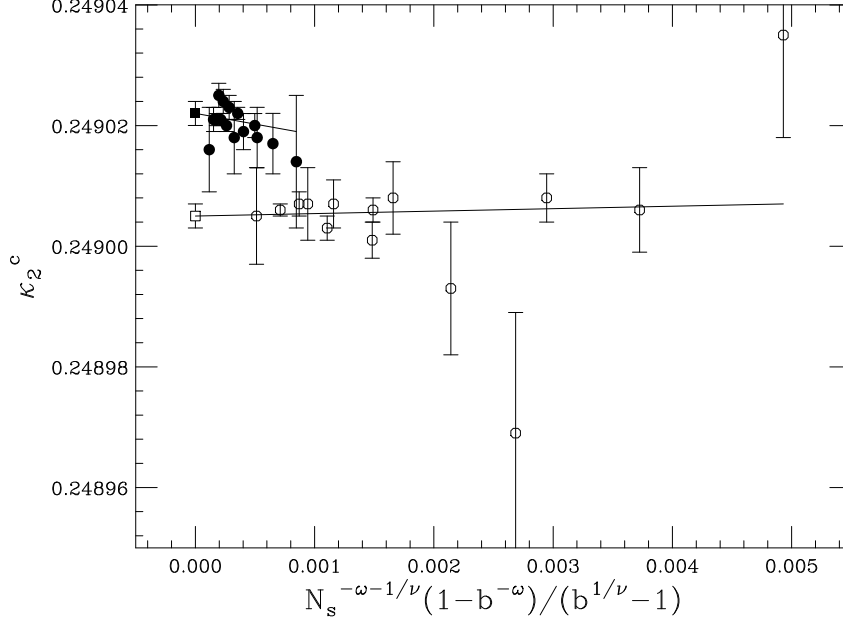


Figure 5: Fits to obtain κ_2^c from the crossings of the Binder cumulants. The results are $\kappa_2^c(T = 0) = 0.249022(2)$ (filled) and $\kappa_2^c(N_t = 2) = 0.249005(2)$ (empty).

4.2 MC Simulation: maximum of the Binder cumulant derivative

Due to the extreme smallness of the shift in the value of κ_2^c , we found it opportune to compute the two values of κ_2^c also in another way, completely independent on the previous one. Indeed, for every value of N_s , one can determine the point $\kappa_2^{**}(N_s)$ where the derivative $\partial U_2 / \partial \kappa_2$ reaches its maximum value, namely C_v . It is well known that, in the limit $N_s \rightarrow \infty$, $\kappa_2^{**}(N_s)$ approaches κ_2^c according to the scaling law (including corrections to scaling):

$$\kappa_2^{**}(N_s) - \kappa_2^c \propto N_s^{-1/\nu}(1 - AN_s^{-\omega}) \quad (21)$$

which allows us to compute κ_2^c . Fig.6 shows the corresponding fits. In this computation, we used the same critical exponents discussed in the previous subsection. We obtained $\kappa_2^c(N_t = 2) = 0.248997(11)$ and $\kappa_2^c(T = 0) = 0.249017(2)$, which are fully compatible with the previous results.

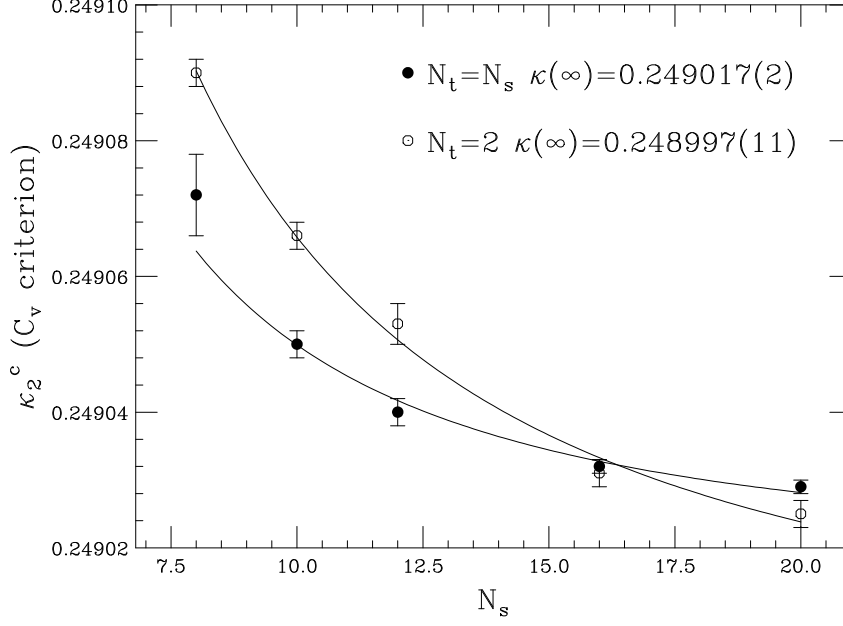


Figure 6: Fits to obtain κ_2^c from the maximum of the Binder cumulant derivative. (The exponents used were $\nu = 0.63, \omega = 0.8$ for $N_t = 2$ and $\nu = 1/2$ for $N_t = N_s$).

As a final check, we computed ν from the maximum of $\partial U_i / \partial \kappa_2$, C_v , which diverges with the lattice size as $N_s^{1/\nu}$. Again, we obtained consistent results, as can be seen in Fig. 7.

In conclusion, this time, differently from what we reported in [15], we found that, when $N_t = 2$, the phase transition of the field ϕ_2 occurs for a value of κ_2 which is *smaller* than at $T = 0$. The two measures of κ_2^c took a long time, because, for the small values of λ_2 and λ used now, the shift of the critical ϕ_2 -line is extremely small and we had to measure both values of κ_2^c with six significant digits, a job that took us months of computer time!

So, the results of our new simulations are favourable to ISB, but cannot be considered as conclusive because they refer to a single value of N_t .

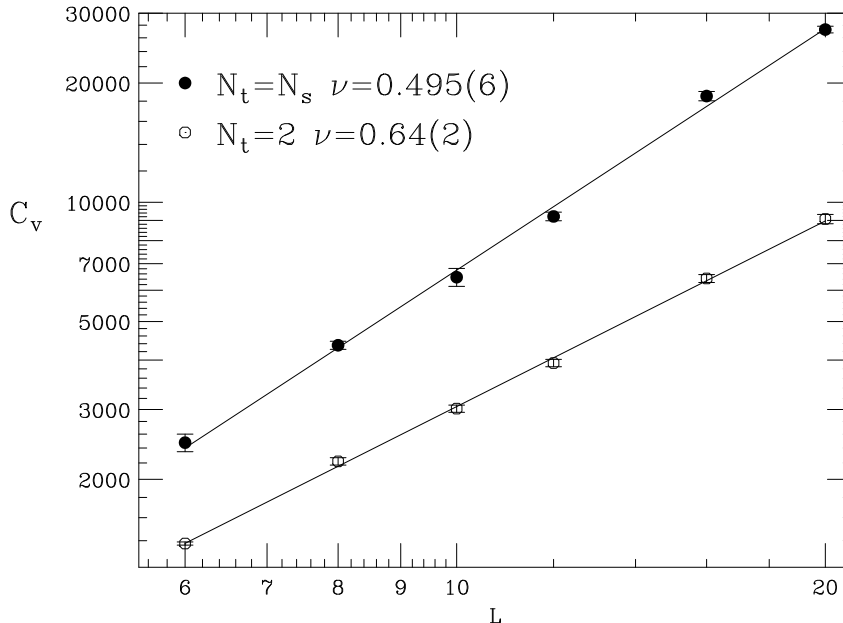


Figure 7: Fits to obtain ν by using the maximum of the Binder cumulant derivative.

5 Comparison with Perturbation theory

As we said in Section 3, simulating only a finite number of N_t values is *per se* not sufficient to claim ISB, for one cannot rule out the possibility that the phenomenon would disappear for larger values of N_t . In fact, the case here is even worse, since we have at our disposal just one value of N_t , $N_t = 2$. In order to increase confidence in our result, we attempted a comparison of our MC data with the perturbative predictions. While the one-loop formulae, eqs.(15), lead to an estimate of the critical temperature far from the MC value, we found a reasonable agreement with the predictions of the gap equations, eqs.(17).

We shall now explain how this comparison was done. Let $p = (\bar{\kappa}_1, \bar{\kappa}_2)$ be the point of the ϕ_2 -critical line for $N_t = 2$, that we identified in our simulations. As we saw, p belongs to the disordered phase of the $T = 0$ phase diagram. If we knew the $T = 0$ values of the renormalized couplings and correlation lengths ξ_1 and ξ_2 (we measure them in units of the lattice

N_s	ξ_1	ξ_2	g_1^R	g_2^R	g^R
6	0.9581(2)	10.78(1)	26(6)	0.0261(9)	-0.35(1)
8	0.9528(3)	11.48(2)	41(11)	0.0296(15)	-0.29(1)

Table 2: $T = 0$ values of the correlation lengths and renormalized couplings for $\kappa_1 = \bar{\kappa}_1 \equiv 0.24$, $\kappa_2 = \bar{\kappa}_2 \equiv 0.2488$ (lattice sizes N_s^4).

spacing) at p , we could get an estimate of the critical inverse temperature N_t^c by solving eq.(15) or eqs.(17), since

$$\left(\frac{T_c}{m_i}\right)^{MC} = \frac{\xi_i}{N_t^c} . \quad (22)$$

If perturbation theory works, we should get for N_t^c a value close to two.

This procedure, even if simple from the conceptual point of view, is difficult to carry out: since p is very close to the critical line of the $T = 0$ phase diagram, the correlation length ξ_2 is huge there (in our case, we estimate it to be larger than 40). Thus a measure of the $T = 0$ renormalized couplings and correlation lengths at p requires 4d-lattices with $N_s \geq 40$, which is just too much for computers available today.

In order to avoid the difficulties connected with exceedingly large correlation lengths for the field ϕ_2 , we measured the renormalized couplings and the correlation lengths (for $T = 0$) at a point $\bar{p} = (\bar{\kappa}_1, \bar{\kappa}_2)$, having the same κ_1 coordinate as p , but with $\bar{\kappa}_2$ slightly less than $\bar{\kappa}_2$, and such that ξ_2 was more or less ten (see Table 2). Because of time limitations, we did not carry out the thermodynamic limit in this series of measures and contented ourselves with simple estimates on rather small lattices. The quantitative considerations that follow have to be taken, then, as simply indicative.

Afterwards, we computed the values of g_1^R , g_2^R , g_R and of ξ_1 and ξ_2 at p , by integrating a set of $(T = 0)$ RGE's in the interval $\bar{\kappa}_2 < \kappa_2 < \bar{\kappa}_2$. We evaluated the β -functions to two-loops order, following a procedure strictly analogous to that discussed in [20]. Of course in our case there was a larger number of β functions to be computed. Since the value of ξ_1 at \bar{p} was only roughly one, we included in our β -functions the corrections to scaling upto one-loop order.

The reader might question the reliability of these perturbative RGE's, since the value of g_1^R at \bar{p} is rather large and quite uncertain. We believe that

this is not a problem, for two reasons. The first one is that the integration interval in κ_2 is very small. The second, and most important, is less obvious: it turns out that g_1^R does not appear in any of the β functions (at least to the two-loops order that we have examined). Its absence can be understood by considering that, since we are moving in the vertical direction in the (κ_1, κ_2) plane, ξ_1 remains finite (and practically constant).

After the numerical integration, it turned out that the running of g_1^R, g_2^R, g^R and ξ_1 in the tiny κ_2 interval considered was completely negligible, while ξ_2 varied from 11 to something like 45. Since the final value of ξ_2 was very sensitive to its initial value, (and practically insensitive to the initial values of the other parameters), we took advantage of our accurate knowledge of $\kappa_2^c(T=0)$, and tuned the initial value of ξ_2 in such a way that κ_2 would approach the MC value of $\kappa_2^c(T=0)$ in the limit $\xi_2 \rightarrow \infty$. The initial value of ξ_2 that we obtained in this way, $\xi_2 = 11.85(5)$, is very close to the MC one, as can be seen from Table 2. The value of ξ_2 at p turned out to be $\xi_2(p) = 45(5)$: the error is basically due to the uncertainty in the MC values of $\kappa_2^c(T=0)$ and $\kappa_2^c(N_t=2)$.

We thus solved eq.(15) and eqs.(17) with respect to N_t^c , using as inputs the values of g_1^R, g_2^R, g^R and ξ_1 given in Table 2 and the above value of $\xi_2(p)$. In this way we obtained the following estimates of N_t^c :

$$N_t^{c(\text{one-loop})} = 4.8(6) , \quad N_t^{c(\text{gap})} = 1.7(2). \quad (23)$$

It is clear that $N_t^{c(\text{one-loop})}$ is far from the MC value $N_t = 2$, while $N_t^{c(\text{gap})}$ is quite close. It seems, then, that resummed perturbation theory provides a rather accurate description of our data.

6 Some comments about recent results on ISB on the Lattice

Recently, there have been two other MC studies of ISB in four dimensions, one by our group [15] and another by Jansen and Laine [16]. While in the first one no sign of ISB was found, the latter authors claimed to have seen clear evidence of ISB and good agreement with resummed perturbation theory. We would like to make here a few comments on these works.

We shall start from our own previous work. The method followed there was very similar to that used in this paper, but the simulations were performed for a different choice of parameters. We think now that the essential reason for the negative result is the fact that the conditions for ISB were not enforced strongly enough. Near the simulation point, the absolute value of g^R was roughly the double of the value of g_2^R : according to the one loop formulae (14), this was more than enough to induce ISB, but nevertheless we did not find it. The reason of the failure is hidden in the gap equations (16). Near our simulation point the correlation length of ϕ_1 was approximately one. This implies that, even at highest temperature (corresponding to $N_t = 2$) and so in the conditions most favourable to ISB, $m_1/T = N_t/\xi_1$ was approximately equal to two. By looking at the gap equations, it is easy to convince oneself that, with a value of $m_1/T \approx 2$, a $|g^R|$ which is twice g_2^R is not enough to induce ISB. The reason is the suppressing factor $h(x_1)$ in front of g^R in the second of eqs.(16): since $x_1 \approx 2$, and $h(2) \approx 0.17$ it is clear that one would have needed a $|g^R|$ at least six times bigger than g_2^R . In order to be on the safe side, in the new simulations presented in this paper, since ξ_1 was still of order one, we selected $|g^R| \approx 10 g_2^R$ which now was large enough.

As for ref.[16], the authors searched for ISB in an $O(4) \times Z_2$ two scalar model. Their approach is very different from ours, and it is useful to briefly review it here. Using perturbation theory (with two-loops accuracy), they computed the relations giving the (bare) lattice parameters, in terms of the renormalized masses and couplings, in the symmetric phase of their model. Then, they performed a set of simulations on lattices with $N_t = 2, 4$ (the few short runs on symmetric lattices, according to the authors, should not be given much importance) for a number of values of the bare couplings, corresponding to decreasing values for the renormalized mass m_1 of the field expected to undergo ISB (ϕ_1), and constant values for the remaining renormalized parameters. In this way they were able to vary the ratio T/m_1 in a large interval, even holding N_t fixed, or varying it very little, by just varying the renormalized mass m_1 . In the simulations they measured the expectation value of the modulus of ϕ_1 , and the results turned out to be fully compatible with the predictions of (resummed) perturbation theory. In particular, the field ϕ_1 appeared to be disordered for values of T/m_1 small enough, and ordered for larger values of that ratio, which is a sign of ISB. Compared with our approach, this procedure presents the advantage of avoiding the very de-

manding task of an accurate determination of the critical point, but on the other side it is necessarily limited to the region of very small couplings, in order for perturbation theory to be applicable. Indeed, in ref.[16], perturbation theory was used not only to compute the relations between the bare and renormalised couplings, as we said above, but also to monitor the behavior of $\langle |\phi_1| \rangle$ in the thermodynamical limit. In view of the long lasting debate, whether ISB and SNR are genuine effects or rather artifacts of perturbation theory, we preferred to pursue the approach presented in this paper which, even if much more expensive, in terms of computer time, does not make any recourse to perturbation theory. As a reward, we were able to probe much larger values of the coupling constants, something which is especially important in view of the applications of ISB to realistic cosmological scenarios, where large couplings in the scalar sector are usually needed in order to enforce ISB, as was pointed out at the end of Sec.2. Moreover, we could also study in detail the nature of the symmetry-breaking high- T phase transition.

7 Conclusions

We have studied ISB at high temperature in four dimensions in a two-scalar ϕ^4 model with $Z_2 \times Z_2$ global symmetry. For fixed values of the coupling constants $\lambda_1, \lambda_2, \lambda$ and of the hopping parameter κ_1 , we measured the critical value of the hopping parameter κ_2 , for which the field ϕ_2 undergoes spontaneous symmetry breaking. This measure was performed both at $T = 0$ and at $T > 0$. It was found that the value of κ_2^c at $T > 0$ is *smaller* than that for $T = 0$, which is a signal of ISB. Due to the extreme smallness of the shift, it was necessary to measure the values of κ_2^c with very high precision. This accuracy was reached via an accurate analysis of FSS on large lattices with very high statistics. The extreme difficulty of the measurements did not allow a scaling analysis of the dependence of the result on the number of sites N_t in the time direction. For this reason, we cannot consider our results as conclusive.

In order to increase our confidence in the result, we compared the MC value of the critical temperature T_c with the theoretical value. While the simple one-loop estimate turned out to be grossly incorrect, we found a reasonable agreement with the value of T_c obtained from the gap equations (17), which result from the resummation of the ring diagrams.

So, in conclusion, we have found some evidence that ISB is possible and that resummed perturbation theory provides a qualitatively correct description of the phenomenon. Nevertheless, we think that our results are not conclusive and that further work is needed before ISB can be claimed with confidence.

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